Transport equations of a consistent description of the kinetics and hydrodynamics of dense quantum systems. I: General approach within the frame of nonequilibrium thermo field dynamics

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Abstract. We present basic equations of nonequilibrium thermo field dynamics of dense quantum systems. A formulation of nonequilibrium thermo field dynamics has been performed using the nonequilibrium statistical operator method by D.N.Zubarev. Generalized transfer and hydrodynamic equations of a consistent description of kinetics and hydrodynamics have been obtained in thermo field representation. To demonstrate how obtained results do work at the description of kinetics and hydrodynamics of a dense nuclear matter we consider quantum system with strongly coupled states.

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1. Introduction

The development of methods for the construction of kinetic and hydrodynamic equations in the theory of nonequilibrium processes for temperature quantum field systems is, in particular, important for the investigation of nonequilibrium properties of a quark-gluon plasma [1, 2, 3, 4, 5] – one of the nuclear matter states which can be created at ultrarelativistic collisions of heavy nuclei [6, 7, 8, 9]. In the studies of nonequilibrium states of quantum field systems, such as a nuclear matter [8, 9], there arises a problem of taking into consideration coupled states. Kinetic and hydrodynamic processes in a hot, compressed nuclear matter, which appears after ultrarelativistic collisions of heavy nuclei or laser thermonuclear synthesis, are mutually connected and we should consider coupled states between nuclons. This is of great importance for the analysis and correlation of final reaction products. Obviously, a nuclon interaction investigation based on a quark-gluon plasma is a sequential microscopic approach to the dynamical description of reactions in a nuclear matter. For the description of kinetic processes in a nuclear matter on the level of model interactions, the Vlasov-Uehling-Uhlenbeck kinetic equation is used. This equation is used mainly in the case of low densities. The

problems of a dense quark-gluon matter were discussed in detail in [8, 9, 10, 11, 12, 13]. As this takes place, its density increases by a factor of ten in the fourth degree and the distance between nuclons in the centre reaches $\sim 10^{-13}$ cm. Such systems are examples of strong both long-range and short-range (nuclear) interactions. There is no small parameter for these systems (density, for example). Nonequilibrium processes have a strongly correlated collective nature. That is why methods which are based on a one-particle description, in particular, on the basis of Boltzmann-like kinetic equations, cannot be used. In addition to high temperature dense quantum systems, there are Bose and Fermi systems at low temperatures with decisive many-particle dissipative correlations. Neither the linear response theory nor Boltzmann-like kinetic equations are sufficient for their description.

Analysis of the problem of a description of kinetic processes in highly nonequilibrium and strongly coupled quantum systems on the basis of the nonequilibrium real-time Green functions technique [14, 15, 16] and the theory in terms of non-Markovian kinetic equations describing memory effects [17, 18, 19] was made in recent paper [10] and then in monograph [20]. It is important to note that in [10] the quantum kinetic equation for a dense and strongly coupled nonequilibrium system was obtained when the parameters of a shortened description included a one-particle Wigner distribution function and an average energy density. On the basis of this approach the quantum Enskog kinetic equation was obtained in [20]. This equation is the quantum analogue of the classical one within the revised Enskog theory [21, 22]. Problems of the construction of kinetic and hydrodynamic equations for highly nonequilibrium and strongly coupled quantum systems were considered based on the nonequilibrium thermo field dynamics in [23, 24, 25, 26, 27, 28]. In particular, a generalized kinetic equation for the average value of the Klimontovich operator was obtained in [24] with the help of the Kawasaki-Gunton projection operator method [29]. The formalism of the nonequilibrium thermo field dynamics was applied to the description of a hydrodynamic state of quantum field systems in paper [25]. Generalized transport equations for nonequilibrium quantum systems, specifically for kinetic and hydrodynamic stages, were obtained in [26] on the basis of the thermo field dynamics conception [30, 31] using the nonequilibrium statistical operator method [20, 32, 33]. In this approach, similarly to [10, 20], the decisive role is that a set of the observed quantities is included in the description of the nonequilibrium process. For these quantities one finds generalized transport equations which should agree with nonequilibrium thermodynamics at controlling the local conservation laws for the particles-number density, momentum and energy. It gives substantial advantages over the nonequilibrium Green function technique [14, 15, 16], which quite well describes excitation spectra, but practically does not describe nonequilibrium thermodynamics, and has problems with the local conservation laws control and the generalized transport coefficients calculation.

In this paper we consider the kinetics and hydrodynamics of highly nonequilibrium and strongly coupled quantum systems using the nonequilibrium thermo field dynamics on the basis of the D.N.Zubarev nonequilibrium statistical operator method [26]. Within

this method we consider a description of the kinetics and hydrodynamics of dense quantum nuclear systems with strongly coupled states. The nonequilibrium thermo field dynamics on the basis of the nonequilibrium statistical operator method constitutes section 2. Thermo field dynamics formalism, superoperators and state vectors in the Liouville thermo field space as well as nonequilibrium statistical operator and projection operators in thermo field space are considered here. A nonequilibrium thermo vacuum state vector is obtained here in view of equations for the generalized hydrodynamics of dense quantum systems. Transport equations of a consistent description of the kinetics and hydrodynamics in thermo field representation are obtained in section 3. We mean that these equations are applied to dense quantum systems where strong coupled states can appear. This item implies, as one of the approaches, to investigate a nonequilibrium nuclear matter [8, 9].

2. Nonequilibrium thermo field dynamics on the basis of Zubarev's method of nonequilibrium statistical operator

Let us consider a quantum system of N interacting bosons or fermions. The Hamiltonian of this system is expressed via creation $a_l^\dagger$ and annihilation a_l operators of the corresponding statistics:

$$H = H(a^\dagger, a). \tag{2.1}$$

Operators $a_l^\dagger$, a_l satisfy the commutation relations:

$$[a_l\,, a_j^\dagger]_\sigma = \delta_{lj}, \quad [a_l\,, a_j]_\sigma = [a_l^\dagger, a_j^\dagger]_\sigma = 0, \tag{2.2}$$

where $[A, B]_\sigma = AB - \sigma BA$, $\sigma = +1$ for bosons and $\sigma = -1$ for fermions.

The nonequilibrium state of such a system is completely described by the nonequilibrium statistical operator $\varrho(t)$. This operator satisfies the quantum Liouville equation

$$\frac{\partial}{\partial t}\varrho(t) - \frac{1}{\mathrm{i}\hbar}[H, \varrho(t)] = 0. \tag{2.3}$$

The nonequilibrium statistical operator $\varrho(t)$ allows us to calculate the average values of operators A

$$\langle A\rangle^t = \mathrm{Sp}\left(A\varrho(t)\right), \tag{2.4}$$

which can be observable quantities describing the nonequilibrium state of the system (for example, a hydrodynamic state is described by the average values of operators of particle number, momentum and energy densities).

The main idea of thermo field dynamics [30, 31] and its nonequilibrium formulation [34, 35, 36] consists in doing the calculation of average values (2.4) with the help of the so-called nonequilibrium thermo vacuum state vector:

$$\langle A\rangle^t = \langle\langle 1|A\varrho(t)\rangle\rangle = \langle\langle 1|\hat{A}|\varrho(t)\rangle\rangle, \tag{2.5}$$

where $\hat{A}$ is a superoperator which acts on state $|\varrho(t)\rangle\rangle$. Nonequilibrium thermo vacuum state vector $|\varrho(t)\rangle\rangle$ satisfies the Schrödinger equation. Starting from equation (2.3), we obtain the relation

$$\frac{\partial}{\partial t}|\varrho(t)\rangle\rangle - \left|\frac{1}{\mathrm{i}\hbar}[H,\varrho(t)]\right\rangle\!\!\!\left.\right\rangle = 0,$$

or, opening commutator,

$$\frac{\partial}{\partial t}|\varrho(t)\rangle\rangle - \frac{1}{\mathrm{i}\hbar}\bar{H}|\varrho(t)\rangle\rangle = 0. \tag{2.6}$$

Here the "total" Hamiltonian $\bar{H}$ reads:

$$\bar{H} = \hat{H} - \tilde{H}, \tag{2.7}$$

and it is known that $\langle\langle 1|\bar{H} = 0$; $\hat{H} = H(\hat{a}^{\dagger}, \hat{a})$, $\tilde{H} = H^{*}(\tilde{a}^{\dagger}, \tilde{a})$ are superoperators which consist of creation and annihilation superoperators without and with a tilde, and which represent the thermal Liouville space [37, 38]. Superoperators $\hat{H}$ and $\tilde{H}$ are defined in accordance with the relations:

$$\begin{aligned} |H\varrho(t)\rangle\rangle &= \hat{H}|\varrho(t)\rangle\rangle, \\ |\varrho(t)H\rangle\rangle &= \tilde{H}|\varrho(t)\rangle\rangle. \end{aligned} \tag{2.8}$$

Hence, it appears that at going from the quantum Liouville equation (2.3) for nonequilibrium statistical operator $\varrho(t)$ to the Schrödinger equation (2.6) for nonequilibrium thermo vacuum state vector $|\varrho(t)\rangle\rangle$, according to (2.5), the number of creation and annihilation operators is doubled. Superoperators $\hat{a}_l^{\dagger}$, $\hat{a}_j$, $\tilde{a}_l^{\dagger}$, $\tilde{a}_j$ satisfy the same commutation relations as for operators $a_l^{\dagger}$, a_j (2.2):

$$\begin{aligned} &[\hat{a}_l, \hat{a}_j^{\dagger}]_{\sigma} = [\tilde{a}_l, \tilde{a}_j^{\dagger}]_{\sigma} = \delta_{lj}, \qquad &[\hat{a}_l, \tilde{a}_j]_{\sigma} = [\hat{a}_l^{\dagger}, \tilde{a}_j^{\dagger}]_{\sigma} = 0, \\ &[\hat{a}_l, \hat{a}_j]_{\sigma} = [\hat{a}_l^{\dagger}, \hat{a}_j^{\dagger}]_{\sigma} = 0, \qquad &[\tilde{a}_l, \tilde{a}_j]_{\sigma} = [\tilde{a}_l^{\dagger}, \tilde{a}_j^{\dagger}]_{\sigma} = 0. \end{aligned} \tag{2.9}$$

Annihilation superoperators $\hat{a}_l$, $\tilde{a}_l$ are defined in accordance with their action on the vacuum state – the supervacuum [37]

$$\hat{a}_l|00\rangle\rangle = \tilde{a}_l|00\rangle\rangle = 0, \tag{2.10}$$

where $|00\rangle\rangle = ||0\rangle\langle 0|\rangle\rangle$ is a supervacuum; and it is known that $\hat{a}_l|0\rangle = a_l|0\rangle = 0$, and $\langle 0|\tilde{a}_l = 0$, i.e. a supervacuum $|00\rangle\rangle$ is an orthogonalized state of two vacuum states $\langle 0|$ and $|0\rangle$. Taking into account commutation relations (2.9), (2.10), one can introduce unit vectors $|1\rangle\rangle = |\sum_l |l\rangle\langle l|\rangle\rangle$ and $\langle\langle 1| = \langle\langle \sum_l |l\rangle\langle l||$ in the following forms:

$$\begin{aligned} |1\rangle\rangle &= \exp\left\{\textstyle\sum_l \hat{a}_l^{\dagger}\tilde{a}_l^{\dagger}\right\}|00\rangle\rangle, \\ \langle\langle 1| &= \langle\langle 00|\exp\left\{\textstyle\sum_l \tilde{a}_l\hat{a}_l\right\}. \end{aligned} \tag{2.11}$$

With the help of these expressions one can find relations between the action of superoperators $\hat{a}_l^{\dagger}$, $\hat{a}_j$, $\tilde{a}_l^{\dagger}$, $\tilde{a}_j$

$$\begin{aligned} \hat{a}_l|1\rangle\rangle &= \tilde{a}_l^{\dagger}|1\rangle\rangle, \qquad &\langle\langle 1|\hat{a}_l^{\dagger} &= \langle\langle 1|\tilde{a}_l, \\ \hat{a}_l^{\dagger}|1\rangle\rangle &= \sigma\tilde{a}_l|1\rangle\rangle, \qquad &\langle\langle 1|\hat{a}_l &= \langle\langle 1|\tilde{a}_l^{\dagger}\sigma. \end{aligned} \tag{2.12}$$

In such a way, in the thermal field dynamics formalism [30, 31] the number of operators is doubled by introducing both without tilde and tildian operators $A(\hat{a}^{\dagger},\hat{a})$, $\tilde{A}(\tilde{a}^{\dagger},\tilde{a})$ for which the following properties take place:

$$\begin{aligned} \widetilde{A_1A_2} &= \tilde{A}_1\tilde{A}_2, \quad \tilde{\tilde{A}} = A, \\ c_1\widetilde{A_1 + c_2}A_2 &= c_1^*\tilde{A}_1 + c_2^*\tilde{A}_2, \\ |A\rangle\rangle &= \hat{A}|1\rangle\rangle, \\ |A_1A_2\rangle\rangle &= \hat{A}_1|A_2\rangle\rangle. \end{aligned} \tag{2.13}$$

Here * denotes a complex conjugation. Some detailed description of the properties of superoperators $\hat{a}_l^{\dagger}$, $\hat{a}_j$, $\tilde{a}_l^{\dagger}$, $\tilde{a}_j$, as well as a thermal Liouville space is given in papers [30, 31, 37, 38].

The nonequilibrium thermo vacuum state vector is normalized

$$\langle\langle 1|\varrho(t)\rangle\rangle = \langle\langle 1|\hat{\varrho}(t)|1\rangle\rangle = 1, \tag{2.14}$$

where $\hat{\varrho}(t)$ is a nonequilibrium statistical superoperator. It depends on superoperators $a_l^{\dagger}$, a_l: $\hat{\varrho}(t) \equiv \varrho\left(\hat{a}^{\dagger},\hat{a};t\right)$, and, it is known that the corresponding tildian superoperator $\tilde{\varrho}(t) \equiv \varrho^{\dagger}(\tilde{a}^{\dagger},\tilde{a};t)$ depends on superoperators $\tilde{a}_l^{\dagger}$, $\tilde{a}_l$.

To solve the Schrödinger equation (2.6) a boundary condition should be given. Following the nonequilibrium statistical operator method [20, 26, 32, 33], let us find a solution to this equation in a form, which depends on time via some set of observable quantities only. It means that this set is sufficient for the description of a nonequilibrium state of a system and does not depend on the initial moment of time. The solution to the Schrödinger equation, which satisfies the following boundary condition

$$|\varrho(t)\rangle\rangle_{t=t_0} = |\varrho_{\mathrm{q}}(t_0)\rangle\rangle, \tag{2.15}$$

reads:

$$|\varrho(t)\rangle\rangle = \exp\left\{(t-t_0)\frac{1}{\mathrm{i}\hbar}\bar{H}\right\}|\varrho_{\mathrm{q}}(t_0)\rangle\rangle. \tag{2.16}$$

We will consider times $t \gg t_0$, when the details of the initial state become inessential. To avoid the dependence on t_0, let us average the solution (2.16) on the initial time moment in the range between t_0 and t and make the limiting transition $t_0 - t \to -\infty$. We will obtain [26]:

$$|\varrho(t)\rangle\rangle = \varepsilon\int_{-\infty}^{0}\mathrm{d}t'\;\mathrm{e}^{\varepsilon t'}\mathrm{e}^{-\frac{1}{\mathrm{i}\hbar}\bar{H}t}|\varrho_{\mathrm{q}}(t+t')\rangle\rangle, \tag{2.17}$$

where $\varepsilon \to +0$ after the thermodynamic limiting transition. Solution (2.17), as it can be shown by its direct differentiation with respect to time t, satisfies the Schrödinger equation with a small source in the right-hand side:

$$\left(\frac{\partial}{\partial t} - \frac{1}{\mathrm{i}\hbar}\bar{H}\right)|\varrho(t)\rangle\rangle = -\varepsilon\Big(|\varrho(t)\rangle\rangle - |\varrho_{\mathrm{q}}(t)\rangle\rangle\Big). \tag{2.18}$$

This source selects retarded solutions which correspond to a shortened description of the nonequilibrium state of a system, $|\varrho_{\mathrm{q}}(t)\rangle\rangle$ is a thermo vacuum quasiequilibrium state vector

$$|\varrho_{\mathrm{q}}(t)\rangle\rangle = \hat{\varrho}_{\mathrm{q}}(t)|1\rangle\rangle. \tag{2.19}$$

Similarly to (2.14), it is normalized by the rule

$$\langle\langle 1|\varrho_{\rm q}(t)\rangle\rangle = \langle\langle 1|\hat{\varrho}_{\rm q}(t)|1\rangle\rangle = 1, \tag{2.20}$$

where $\hat{\varrho}_{\rm q}(t)$ is a quasiequilibrium statistical superoperator. The quasiequilibrium thermo vacuum state vector of a system is introduced in the following way. Let $\langle p_n\rangle^t = \langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle$ be a set of observable quantities which describe the nonequilibrium state of a system. p_n are operators which consist of the creation and annihilation operators defined in (2.2). Quasiequilibrium statistical operator $\varrho_{\rm q}(t)$ is defined from the condition of informational entropy $S_{\rm inf}$ extremum (maximum) at additional conditions of prescribing the average values $\langle p_n\rangle^t$ and conservation of normalization condition (2.20) [20, 33]:

$$\begin{aligned} \varrho_{\rm q}(t) &= \exp\left\{-\Phi(t) - \sum\nolimits_n F_n^*(t)p_n\right\}, \\ \Phi(t) &= \ln \mathrm{Sp}\exp\left\{-\sum\nolimits_n F_n^*(t)p_n\right\}, \end{aligned} \tag{2.21}$$

where $\Phi(t)$ is the Massieu-Planck functional. A summation on n can designate a sum with respect to the wave-vector $\boldsymbol{k}$, the kind of particles and a line of other quantum numbers, a spin for example. Parameters $F_n(t)$ are defined from the conditions of self-consistency:

$$\langle p_n\rangle^t = \langle p_n\rangle^t_{\rm q}, \qquad \langle\ldots\rangle^t_{\rm q} = \mathrm{Sp}\Big(\ldots\varrho_{\rm q}(t)\Big). \tag{2.22}$$

According to (2.5), let us write these conditions of self-consistency in the following form:

$$\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle = \langle\langle 1|\hat{p}_n|\varrho_{\rm q}(t)\rangle\rangle. \tag{2.23}$$

Taking into account behaviours (2.13), we have:

$$|\varrho_{\rm q}(t)\rangle\rangle = \hat{\varrho}_{\rm q}(t)|1\rangle\rangle = \tilde{\varrho}^\dagger_{\rm q}(t)|1\rangle\rangle, \tag{2.24}$$

where

$$\begin{aligned} \hat{\varrho}_{\rm q}(t) &= \exp\left\{-\Phi(t) - \sum\nolimits_n F_n^*(t)\hat{p}_n\right\}, \\ \tilde{\varrho}^\dagger_{\rm q}(t) &= \exp\left\{-\Phi(t) - \sum\nolimits_n F_n(t)\tilde{p}_n\right\} \end{aligned} \tag{2.25}$$

are quasiequilibrium statistical superoperators which contain superoperators $\hat{p}_n$ and $\tilde{p}_n$, correspondingly:

$$\begin{aligned} \hat{p}_n &= p_n(\hat{a}^\dagger, \hat{a}), \\ \tilde{p}_n &= p_n^*(\tilde{a}^\dagger, \tilde{a}). \end{aligned} \tag{2.26}$$

If self-consistency condition (2.23) realizes, we shall have the following relations (at fixed corresponding parameters):

$$\begin{aligned} \frac{\delta\Phi(t)}{\delta F_n^*(t)} &= \langle\langle 1|\hat{p}_n|\varrho_{\rm q}(t)\rangle\rangle = \langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle, \\ \frac{\delta\Phi(t)}{\delta F_n(t)} &= \langle\langle 1|\tilde{p}_n|\varrho_{\rm q}(t)\rangle\rangle = \langle\langle 1|\tilde{p}_n|\varrho(t)\rangle\rangle. \end{aligned} \tag{2.27}$$

Relations (2.27) show that parameters $F_n^*(t)$, $F_n(t)$ are conjugated to the averages $\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle$ and $\langle\langle 1|\tilde{p}_n|\varrho(t)\rangle\rangle$, correspondingly. On the other hand, with the help of $|\varrho_{\rm q}(t)\rangle\rangle$ and self-consistency conditions (2.23) we can define the entropy of the system state:

$$S(t) = -\langle\langle 1|(\,\ln\varrho_{\rm q}(t))\varrho_{\rm q}(t)\rangle\rangle = \Phi(t) + \sum\nolimits_n F_n^*(t)\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle. \tag{2.28}$$

The physical meaning of parameters $F_n^*(t)$ can be obtained now on the basis of the previous relation:

$$F_n^*(t) = \frac{\delta S(t)}{\delta\langle\langle 1|\hat{p}_n|\varrho(t)}. \tag{2.29}$$

Now the auxiliary quasiequilibrium thermo vacuum state vector $|\varrho_{\rm q}(t)\rangle\rangle$ is defined. Let us represent solution (2.17) of the Schrödinger equation (2.6) in a form which is more convenient for the construction of transport equations for averages $\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle$. We shall start from the Schrödinger equation with a small source (2.18). Let us rebuild this equation by introducing $\triangle|\varrho(t)\rangle\rangle = |\varrho(t)\rangle\rangle - |\varrho_{\rm q}(t)\rangle\rangle$:

$$\left(\frac{\partial}{\partial t} - \frac{1}{{\rm i}\hbar}\bar{H} + \varepsilon\right)\triangle|\varrho(t)\rangle\rangle = -\left(\frac{\partial}{\partial t} - \frac{1}{{\rm i}\hbar}\bar{H}\right)|\varrho_{\rm q}(t)\rangle\rangle. \tag{2.30}$$

The calculation of time derivation of $|\varrho_{\rm q}(t)\rangle\rangle$ in the right-hand side of equation (2.30) is equivalent to the introduction of the Kawasaki-Gunton projection operator $\mathscr{P}_{\rm q}(t)$ [26] in thermo field representation:

$$\frac{\partial}{\partial t}|\varrho_{\rm q}(t)\rangle\rangle = \mathscr{P}_{\rm q}(t)\frac{1}{{\rm i}\hbar}\bar{H}|\varrho(t)\rangle\rangle, \tag{2.31}$$

$$\begin{aligned} &\mathscr{P}_{\rm q}(t)(|\ldots\rangle\rangle) = |\varrho_{\rm q}(t)\rangle\rangle + \\ &\sum_n \frac{\delta|\varrho_{\rm q}(t)\rangle\rangle}{\delta\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle}\langle\langle 1|\hat{p}_n|\ldots\rangle\rangle - \sum_n \frac{\delta|\varrho_{\rm q}(t)\rangle\rangle}{\delta\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle}\langle\langle 1|\hat{p}_n|\ldots\rangle\rangle\langle\langle 1|\ldots\rangle\rangle. \end{aligned} \tag{2.32}$$

Projection operator $\mathscr{P}_{\rm q}(t)$ acts on state vectors $|\ldots\rangle\rangle$ only and has all the operator properties:

$$\begin{aligned} \mathscr{P}_{\rm q}(t)|\varrho(t')\rangle\rangle &= |\varrho_{\rm q}(t)\rangle\rangle, \\ \mathscr{P}_{\rm q}(t)|\varrho_{\rm q}(t')\rangle\rangle &= |\varrho_{\rm q}(t)\rangle\rangle, \\ \mathscr{P}_{\rm q}(t)\mathscr{P}_{\rm q}(t') &= \mathscr{P}_{\rm q}(t). \end{aligned}$$

Taking into account condition $\mathscr{P}_{\rm q}(t)\frac{1}{{\rm i}\hbar}\bar{H}\;\triangle|\varrho(t)\rangle\rangle = 0$, one may rewrite equation (2.30), after simple reductions, in a form:

$$\left(\frac{\partial}{\partial t} - \Big(1 - \mathscr{P}_{\rm q}(t)\Big)\frac{1}{{\rm i}\hbar}\bar{H} + \varepsilon\right)\triangle|\varrho(t)\rangle\rangle = \Big(1 - \mathscr{P}_{\rm q}(t)\Big)\frac{1}{{\rm i}\hbar}\bar{H}|\varrho_{\rm q}(t)\rangle\rangle. \tag{2.33}$$

The formal solution to this equation reads:

$$\triangle|\varrho(t)\rangle\rangle = \int_{-\infty}^{t}{\rm d}t'\;{\rm e}^{\varepsilon(t'-t)}T(t,t')\Big(1 - \mathscr{P}_{\rm q}(t')\Big)\frac{1}{{\rm i}\hbar}\bar{H}|\varrho_{\rm q}(t')\rangle\rangle, \qquad \text{or}$$

$$|\varrho(t)\rangle\rangle = |\varrho_{\mathrm{q}}(t)\rangle\rangle + \int_{-\infty}^{t} \mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)} T(t,t') \Big(1 - \mathscr{P}_{\mathrm{q}}(t')\Big) \frac{1}{\mathrm{i}\hbar} \bar{H} |\varrho_{\mathrm{q}}(t')\rangle\rangle, \quad (2.34)$$

where

$$T(t,t') = \exp_{+}\left\{ \int_{t'}^{t} \mathrm{d}t'\ \Big(1 - \mathscr{P}_{\mathrm{q}}(t')\Big) \frac{1}{\mathrm{i}\hbar} \bar{H} \right\} \quad (2.35)$$

is an evolution operator with projection consideration, and $\exp_{+}$ is an ordered exponent. Then, let us consider expression $\Big(1 - \mathscr{P}_{\mathrm{q}}(t')\Big) \frac{1}{\mathrm{i}\hbar} \bar{H} |\varrho_{\mathrm{q}}(t)\rangle\rangle$ in the right-hand side of (2.30). The action of $\frac{1}{\mathrm{i}\hbar}\bar{H}$ and $\Big(1 - \mathscr{P}_{\mathrm{q}}(t')\Big)$ on $|\varrho_{\mathrm{q}}(t)\rangle\rangle$ can be represented in the form:

$$\Big(1 - \mathscr{P}_{\mathrm{q}}(t')\Big) \frac{1}{\mathrm{i}\hbar} \bar{H} |\varrho_{\mathrm{q}}(t)\rangle\rangle = \sum_{n} F_n^{*}(t) \left| \int_0^1 \mathrm{d}\tau\ \varrho_{\mathrm{q}}^{\tau}(t) \Big(1 - \mathscr{P}(t')\Big) \dot{p}_n \varrho_{\mathrm{q}}^{1-\tau}(t) \right\rangle\!\!\!\Big\rangle, \quad (2.36)$$

where $\dot{p}_n$ and $\mathscr{P}(t)$ read:

$$\dot{p}_n = -\frac{1}{\mathrm{i}\hbar}[H, p_n], \quad (2.37)$$

$$\mathscr{P}(t) p = \langle\langle 1|\hat{p}|\varrho_{\mathrm{q}}(t)\rangle\rangle + \sum_{n} \frac{\delta\langle\langle 1|\hat{p}|\varrho_{\mathrm{q}}(t)\rangle\rangle}{\delta\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle} \Big(p_n - \langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle\Big). \quad (2.38)$$

Here $\mathscr{P}(t)$ is a generalized Mori projection operator in thermo field representation. It acts on operators and has the following properties:

$$\begin{aligned} \mathscr{P}(t) p_n &= p_n, \\ \mathscr{P}(t)\mathscr{P}(t') &= \mathscr{P}(t). \end{aligned} \quad (2.39)$$

Let us substitute now (2.36) into (2.34) and, as a result, we will obtain an expression for the nonequilibrium thermo vacuum state of a system:

$$|\varrho(t)\rangle\rangle = |\varrho_{\mathrm{q}}(t)\rangle\rangle + \sum_{n} \int_{-\infty}^{t} \mathrm{d}t' \mathrm{e}^{\varepsilon(t'-t)} T(t,t') \left| \int_0^1 \mathrm{d}\tau \varrho_{\mathrm{q}}^{\tau}(t') J_n(t') \varrho_{\mathrm{q}}^{1-\tau}(t') \right\rangle\!\!\!\Big\rangle F_n^{*}(t'). \quad (2.40)$$

$$J_n(t) = \Big(1 - \mathscr{P}(t)\Big) \dot{p}_n \quad (2.41)$$

are generalized flows.

Let us obtain now transport equations for averages $\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle$ in thermo field representation with the help of nonequilibrium thermo vacuum state vector $|\varrho(t)\rangle\rangle$ (2.40). To achieve this we will use the equality

$$\frac{\partial}{\partial t}\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle = \langle\langle 1|\dot{\hat{p}}_n|\varrho(t)\rangle\rangle = \langle\langle 1|\dot{\hat{p}}_n|\varrho_{\mathrm{q}}(t)\rangle\rangle + \langle\langle J_n(t)|\varrho(t)\rangle\rangle. \quad (2.42)$$

By making use of $|\varrho(t)\rangle\rangle$ in (2.40) in averaging the last term, we obtain transport equations for $\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle$:

$$\begin{aligned} &\frac{\partial}{\partial t}\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle = \langle\langle 1|\dot{\hat{p}}_n|\varrho_{\mathrm{q}}(t)\rangle\rangle + \quad (2.43) \\ &\sum_{n'} \int_{-\infty}^{t} \mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)} \Big\langle\!\!\!\Big\langle J_n(t) T(t,t') \left| \int_0^1 \mathrm{d}\tau\ \varrho_{\mathrm{q}}^{\tau}(t') J_{n'}(t') \varrho_{\mathrm{q}}^{1-\tau}(t') \right\rangle\!\!\!\Big\rangle\ F_{n'}^{*}(t'), \end{aligned}$$

where $\dot{\hat{p}}_n = -\frac{1}{\mathrm{i}\hbar}\bar{H}\hat{p}_n$. Relations (2.43) are treated as a general form of transport equations for average values of a shortened description. These equations can be applied to completely actual problems.

3. Transport equations of dense quantum systems with coupled states

We will consider a quantum field system in which coupled states can appear between the particles. Let us introduce annihilation and creation operators of a coupled state $(A\alpha)$ with A-particle:

$$\begin{aligned} a_{A\alpha}(\boldsymbol{p}) &= \sum_{1,\ldots,A} \Psi_{A\alpha\boldsymbol{p}}(1,\ldots,A)a(1)\ldots a(A), \\ a^{\dagger}_{A\alpha}(\boldsymbol{p}) &= \sum_{1,\ldots,A} \Psi^{*}_{A\alpha\boldsymbol{p}}(1,\ldots,A)a^{\dagger}(1)\ldots a^{\dagger}(A), \end{aligned} \tag{3.1}$$

where $\Psi_{A\alpha\boldsymbol{p}}(1,\ldots,A)$ is a self-function of the A-particle coupled state, α denotes internal quantum numbers (spin, etc.), $\boldsymbol{p}$ is a particle momentum, the sum covers the particles. Annihilation and creation operators $a(j)$ and $a^{\dagger}(j)$ satisfy the following commutation relations:

$$[a(l), a^{\dagger}(j)]_{\sigma} = \delta_{l,j}, \qquad [a(l), a(j)]_{\sigma} = [a^{\dagger}(l), a^{\dagger}(j)]_{\sigma} = 0, \tag{3.2}$$

where σ-commutator is determined by $[a,b]_{\sigma} = ab - \sigma ba$ with $\sigma = \pm 1$: $+1$ for bosons and -1 for fermions.

The Hamiltonian of such a system can be written in the form:

$$\begin{aligned} H = {}& \sum_{A,\alpha} \int \frac{\mathrm{d}\boldsymbol{p}\mathrm{d}\boldsymbol{q}}{(2\pi\hbar)^6} \frac{p^2}{2m_A} a^{\dagger}_{A\alpha}\left(\boldsymbol{p} - \frac{\boldsymbol{q}}{2}\right) a_{A\alpha}\left(\boldsymbol{p} + \frac{\boldsymbol{q}}{2}\right) + \\ & \frac{1}{2}\sum_{A,B}\sum_{\alpha,\beta} \int \frac{\mathrm{d}\boldsymbol{p}\mathrm{d}\boldsymbol{p}'\mathrm{d}\boldsymbol{q}}{(2\pi\hbar)^9} V_{AB}(\boldsymbol{q}) a^{\dagger}_{A\alpha}\left(\boldsymbol{p} + \frac{\boldsymbol{q}-\boldsymbol{p}'}{2}\right) \hat{n}_{B\beta}(\boldsymbol{q}) a_{A\alpha}\left(\boldsymbol{p} - \frac{\boldsymbol{q}-\boldsymbol{p}'}{2}\right), \end{aligned} \tag{3.3}$$

where $V_{AB}(\boldsymbol{q})$ is interaction energy between A- and B-particle coupled states, $\boldsymbol{q}$ is a wavevector. Annihilation and creation operators $a_{A\alpha}(\boldsymbol{p})$ and $a^{\dagger}_{A\alpha}(\boldsymbol{p})$ satisfy the following commutation relations:

$$\begin{aligned} &[a_{A\alpha}(\boldsymbol{p}), a^{\dagger}_{B\beta}(\boldsymbol{p}')]_{\sigma} = \delta_{A,B}\delta_{\alpha,\beta}\delta(\boldsymbol{p} - \boldsymbol{p}'), \\ &[a_{A\alpha}(\boldsymbol{p}), a_{B\beta}(\boldsymbol{p}')]_{\sigma} = [a^{\dagger}_{A\alpha}(\boldsymbol{p}), a^{\dagger}_{B\beta}(\boldsymbol{p}')]_{\sigma} = 0. \end{aligned} \tag{3.4}$$

$\hat{n}_{B\beta}(\boldsymbol{q})$ in (3.3) is a Fourier transform of the B-particle density operator:

$$\hat{n}_{B\beta}(\boldsymbol{q}) = \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3}\, a^{\dagger}_{\boldsymbol{p}-\frac{\boldsymbol{q}}{2}} a_{\boldsymbol{p}+\frac{\boldsymbol{q}}{2}}.$$

As parameters $\langle\langle 1|\hat{p}_n|\varrho(t)\rangle\rangle$ of a shortened description for the consistent description of the kinetics and hydrodynamics of a system, where coupled states between the particles can appear, let us choose nonequilibrium distribution functions of A-particle coupled states in thermo field representation

$$\langle\langle 1|\hat{n}_{A\alpha}(\boldsymbol{r},\boldsymbol{p})|\varrho(t)\rangle\rangle = f_{A\alpha}(\boldsymbol{r},\boldsymbol{p};t) = f_{A\alpha}(x;t), \quad x = \{\boldsymbol{r},\boldsymbol{p}\}, \tag{3.5}$$

here $f_{A\alpha}(x;t)$ is a Wigner function of the A-particle coupled state where

$$\hat{n}_{A\alpha}(\boldsymbol{r},\boldsymbol{p}) \equiv \hat{n}_{A\alpha}(x) = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi\hbar)^3}\, \mathrm{e}^{-\frac{1}{\mathrm{i}\hbar}\boldsymbol{q}\cdot\boldsymbol{r}} \hat{a}^{\dagger}_{A\alpha}\left(\boldsymbol{p}-\frac{\boldsymbol{q}}{2}\right) \hat{a}_{A\alpha}\left(\boldsymbol{p}+\frac{\boldsymbol{q}}{2}\right) \tag{3.6}$$

is the Klimontovich density operator; and the average value of the total energy density operator

$$\langle\langle 1|\hat{H}(\boldsymbol{r})|\varrho(t)\rangle\rangle = \langle\langle 1|H(\boldsymbol{r})\varrho(t)\rangle\rangle. \tag{3.7}$$

By this $\int \mathrm{d}\boldsymbol{r}\; H(\boldsymbol{r}) = H$, $\hat{H}(\boldsymbol{r})$ is a superoperator of the total energy density which is constructed on annihilation and creation superoperators $\hat{a}\ A\alpha(\boldsymbol{p})$ and $\hat{a}^{\dagger}_{A\alpha}(\boldsymbol{p})$. The latter satisfy commutation relations (3.4). Following [26] and (2.21), one can rewrite quasiequilibrium statistical operator $\hat{\varrho}_{\mathrm{q}}(t)$, $|\varrho_{\mathrm{q}}(t)\rangle\rangle = \hat{\varrho}_{\mathrm{q}}(t)|1\rangle\rangle$ for the mentioned parameters of a shortened description in the form:

$$\hat{\varrho}_{\mathrm{q}}(t) = \exp\left\{-\Phi^*(t) - \int \mathrm{d}\boldsymbol{r}\; \beta(\boldsymbol{r};t)\left(\hat{H}(\boldsymbol{r}) - \sum_{A,\alpha}\int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3}\, \mu_{A\alpha}(x;t)\hat{n}_{A\alpha}(x)\right)\right\}, \tag{3.8}$$

where Lagrange multipliers $\beta(\boldsymbol{r};t)$ and $\mu_{A\alpha}(x;t)$ can be found from the self-consistency conditions (2.22), correspondingly:

$$\langle\langle 1|\hat{H}(\boldsymbol{r})|\varrho(t)\rangle\rangle \quad = \langle\langle 1|\hat{H}(\boldsymbol{r})|\varrho_{\mathrm{q}}(t)\rangle\rangle, \tag{3.9}$$

$$\langle\langle 1|\hat{n}_{A\alpha}(x)|\varrho(t)\rangle\rangle = \langle\langle 1|\hat{n}_{A\alpha}(x)|\varrho_{\mathrm{q}}(t)\rangle\rangle, \tag{3.10}$$

$\Phi^*(t)$ is the Massieu-Planck functional and it can be defined from the normalization condition (2.20):

$$\Phi^*(t) = \ln \left\langle\!\!\left\langle 1 \left| \exp\left\{-\int \mathrm{d}\boldsymbol{r}\; \beta(\boldsymbol{r};t)\left(\hat{H}(\boldsymbol{r}) - \sum_{A,\alpha}\int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3}\mu_{A\alpha}(x;t)\hat{n}_{A\alpha}(x)\right)\right\} \right.\right\rangle\!\!\right\rangle. \tag{3.11}$$

Using now the general structure of nonequilibrium thermo field dynamics (2.30)–(2.43), one can obtain a set of generalized transport equations for A-particle Wigner distribution functions and the average interaction energy:

$$\begin{aligned}\frac{\partial}{\partial t}\langle\langle 1|\hat{n}_{A\alpha}(x)|\varrho(t)\rangle\rangle &= \langle\langle 1|\dot{\hat{n}}_{A\alpha}(x)|\varrho_{\mathrm{q}}(t)\rangle\rangle + \int \mathrm{d}\boldsymbol{r}' \int_{-\infty}^{t} \mathrm{d}t'\; \mathrm{e}^{\varepsilon(t'-t)}\varphi^{A\alpha}_{nH}(x,\boldsymbol{r}';t,t')\beta(\boldsymbol{r}';t') \\ &+ \sum_{B,\beta}\int \mathrm{d}x' \int_{-\infty}^{t} \mathrm{d}t'\; \mathrm{e}^{\varepsilon(t'-t)}\varphi^{A\alpha B\beta}_{nn}(x,x';t,t')\beta(\boldsymbol{r}';t')\mu_{B\beta}(x';t'),\end{aligned} \tag{3.12}$$

$$\begin{aligned}\frac{\partial}{\partial t}\langle\langle 1|\hat{H}(\boldsymbol{r})|\varrho(t)\rangle\rangle &= \langle\langle 1|\dot{\hat{H}}(\boldsymbol{r})|\varrho_{\mathrm{q}}(t)\rangle\rangle + \int \mathrm{d}\boldsymbol{r}' \int_{-\infty}^{t} \mathrm{d}t'\; \mathrm{e}^{\varepsilon(t'-t)}\varphi_{HH}(\boldsymbol{r},\boldsymbol{r}';t,t')\beta(\boldsymbol{r}';t') \\ &+ \sum_{B,\beta}\int \mathrm{d}x' \int_{-\infty}^{t} \mathrm{d}t'\; \mathrm{e}^{\varepsilon(t'-t)}\varphi^{B\beta}_{Hn}(\boldsymbol{r},x';t,t')\beta(\boldsymbol{r}';t')\mu_{B\beta}(x';t'),\end{aligned} \tag{3.13}$$

where $x' = \{\boldsymbol{r}', \boldsymbol{p}'\}$, $dx' = (2\pi\hbar)^{-3} d\boldsymbol{r}'\, d\boldsymbol{p}'$. Here

$$\varphi_{nn}^{\substack{A\alpha\\B\beta}}(x, x'; t, t') = \left\langle\!\!\left\langle 1 \left| \hat{J}_{n_{A\alpha}}(x, t) T(t, t') \right| \int_0^1 \mathrm{d}\tau\ \varrho_{\mathrm{q}}^{\tau}(t') J_{n_{B\beta}}(x'; t') \varrho_{\mathrm{q}}^{1-\tau}(t') \right\rangle\!\!\right\rangle, \tag{3.14}$$

$$\varphi_{nH}^{A\alpha}(x, \boldsymbol{r}'; t, t') = \left\langle\!\!\left\langle 1 \left| \hat{J}_{n_{A\alpha}}(x, t) T(t, t') \right| \int_0^1 \mathrm{d}\tau\ \varrho_{\mathrm{q}}^{\tau}(t') J_H(\boldsymbol{r}'; t') \varrho_{\mathrm{q}}^{1-\tau}(t') \right\rangle\!\!\right\rangle, \tag{3.15}$$

$$\varphi_{Hn}^{B\beta}(\boldsymbol{r}', x'; t, t') = \left\langle\!\!\left\langle 1 \left| \hat{J}_H(\boldsymbol{r}, t) T(t, t') \right| \int_0^1 \mathrm{d}\tau\ \varrho_{\mathrm{q}}^{\tau}(t') J_{n_{B\beta}}(x'; t') \varrho_{\mathrm{q}}^{1-\tau}(t') \right\rangle\!\!\right\rangle, \tag{3.16}$$

$$\varphi_{HH}(\boldsymbol{r}, \boldsymbol{r}'; t, t') = \left\langle\!\!\left\langle 1 \left| \hat{J}_H(\boldsymbol{r}, t) T(t, t') \right| \int_0^1 \mathrm{d}\tau\ \varrho_{\mathrm{q}}^{\tau}(t') J_H(\boldsymbol{r}'; t') \varrho_{\mathrm{q}}^{1-\tau}(t') \right\rangle\!\!\right\rangle, \tag{3.17}$$

are generalized transport cores which describe dissipative processes. In these formulae J are generalized flows:

$$\begin{aligned} J_H(\boldsymbol{r}; t) &= \Big(1 - \mathscr{P}(t')\Big) \dot{H}(\boldsymbol{r}), & \dot{H}(\boldsymbol{r}) &= -\frac{1}{\mathrm{i}\hbar}[H, H(\boldsymbol{r})], \\ J_{n_{A\alpha}}(\boldsymbol{r}, \boldsymbol{p}; t) &= \Big(1 - \mathscr{P}(t')\Big) \dot{n}_{A\alpha}(x), & \dot{n}_{A\alpha}(\boldsymbol{r}, \boldsymbol{p}) &= -\frac{1}{\mathrm{i}\hbar}[H, n_{A\alpha}(x)], \end{aligned} \tag{3.18}$$

$\mathscr{P}(t)$ is a generalized Mori projection operator in thermo field representation. It acts on operators

$$\begin{aligned} \mathscr{P}(t) P = \langle\langle|\hat{P}|\varrho_{\mathrm{q}}(t)\rangle\rangle + \int \mathrm{d}\boldsymbol{r}\ \frac{\delta\langle\langle 1|\hat{P}|\varrho_{\mathrm{q}}(t)\rangle\rangle}{\delta\langle\langle 1|\hat{H}(\boldsymbol{r})|\varrho(t)\rangle\rangle} \Big(H(\boldsymbol{r}) - \langle\langle 1|\hat{H}(\boldsymbol{r})|\varrho(t)\rangle\rangle \Big) \\ + \sum_{A,\alpha} \int \frac{\mathrm{d}\boldsymbol{r}\, \mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3} \frac{\delta\langle\langle 1|\hat{P}|\varrho_{\mathrm{q}}(t)\rangle\rangle}{\delta\langle\langle 1|\hat{n}_{A\alpha}(x)|\varrho(t)\rangle\rangle} \Big(n_{A\alpha}(x) - \langle\langle 1|\hat{n}_{A\alpha}(x)|\varrho(t)\rangle\rangle \Big) \end{aligned} \tag{3.19}$$

and has all the properties of a projection operator:

$$\begin{aligned} \mathscr{P}(t) H(\boldsymbol{r}) &= H(\boldsymbol{r}), & \mathscr{P}(t)\mathscr{P}(t') &= \mathscr{P}(t), \\ \mathscr{P}(t) n_{A\alpha}(\boldsymbol{r}, \boldsymbol{p}) &= n_{A\alpha}(\boldsymbol{r}, \boldsymbol{p}), & \Big(1 - \mathscr{P}(t)\Big)\mathscr{P}(t) &= 0. \end{aligned}$$

The obtained transport equations have the general meaning and can describe both weakly and strongly nonequilibrium processes of a quantum system with taking into consideration coupled states. In a low density quantum field Bose- or Fermi-system the influence of the average value of interaction energy is substantially smaller than the average kinetic energy, and coupled states between the particles are absent. In such a case the set of transport equations (3.12), (3.13) is simplified. It transforms into a kinetic equation [26] in thermo field representation for the average value of the Klimontovich operator $\langle\langle 1|\hat{n}(x)|\varrho(t)\rangle\rangle$:

$$\begin{aligned} &\frac{\partial}{\partial t}\langle\langle 1|\hat{n}_{\boldsymbol{k}}(\boldsymbol{p})|\varrho(t)\rangle\rangle = \langle\langle 1|\dot{\hat{n}}_{\boldsymbol{k}}(\boldsymbol{p})|\varrho_{\mathrm{q}}(t)\rangle\rangle + \\ &\sum_{\boldsymbol{g}} \int \mathrm{d}\boldsymbol{p}' \int_{-\infty}^{t} \mathrm{d}t'\ \mathrm{e}^{\varepsilon(t'-t)} \left\langle\!\!\left\langle J_n(\boldsymbol{k}; t) \left| T(t, t') \right| \int_0^1 \mathrm{d}\tau\ \varrho_{\mathrm{q}}^{\tau}(t') J_n(\boldsymbol{g}; t') \varrho_{\mathrm{q}}^{1-\tau}(t') \right\rangle\!\!\right\rangle b_{-\boldsymbol{g}}(\boldsymbol{p}'; t'). \end{aligned}$$

Using the projection operators method, this equation was obtained in [23].

In the next step we will construct such annihilation and creation superoperators, for which the quasiequilibrium thermo vacuum state vector is a vacuum state. Analysing

the structure of quasiequilibrium statistical superoperator (3.8), one can mark out some part which would correspond to the system of noninteracting quantum A-particles. Let us write $\hat{\varrho}_{\mathrm{q}}(t)$ in an evident form and separate terms which are connected with the interaction energy between the particles:

$$\hat{\varrho}_{\mathrm{q}}(t) = \exp\Bigg\{-\Phi^*(t) - \int \mathrm{d}\boldsymbol{r}\ \beta(\boldsymbol{r};t) \times \tag{3.20}$$
$$\sum_{A,\alpha}\int \frac{\mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3}\left[\frac{\boldsymbol{p}^2}{2m_A}\hat{n}_{A\alpha}(x) - \mu_{A\alpha}(x;t)\hat{n}_{A\alpha}(x)\right] - \int \mathrm{d}\boldsymbol{r}\beta(\boldsymbol{r};t)\hat{H}_{\mathrm{int}}(\boldsymbol{r})\Bigg\}.$$

Using operator equality (A and B are some operators)

$$\mathrm{e}^{A+B} = \left[1 + \int_0^1 \mathrm{d}\tau\ \mathrm{e}^{\tau(A+B)}\, B\, \mathrm{e}^{-\tau(A+B)}\right]\mathrm{e}^{A},$$

the relation for $\hat{\varrho}_{\mathrm{q}}(t)$ can be rewritten in the following form:

$$\hat{\varrho}_{\mathrm{q}}(t) = \left[1 - \int \mathrm{d}\boldsymbol{r}\ \beta(\boldsymbol{r};t)\int_0^1 \mathrm{d}\tau\ \hat{\varrho}_{\mathrm{q}}^{\tau}(t)\hat{H}_{\mathrm{int}}(\boldsymbol{r})(\hat{\varrho}_{\mathrm{q}}(t))^{-\tau}\right]\hat{\varrho}_{\mathrm{q}}^{0}(t), \tag{3.21}$$

where

$$\hat{\varrho}_{\mathrm{q}}^{0}(t) = \exp\left\{\Phi(t) - \int \mathrm{d}\boldsymbol{r}\ \beta(\boldsymbol{r};t)\sum_{A,\alpha}\int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3} b_{A\alpha}(x;t)\hat{n}_{A\alpha}(x)\right\}, \tag{3.22}$$

$$b_{A\alpha}(x;t) = \left[\frac{\boldsymbol{p}^2}{2m_A}\hat{n}_{A\alpha}(x) - \mu_{A\alpha}(x;t)\hat{n}_{A\alpha}(x)\right]. \tag{3.23}$$

Quasiequilibrium statistical superoperator $\hat{\varrho}_{\mathrm{q}}^{0}(t)$ is bilinear on annihilation and creation superoperators $\hat{a}_{A\alpha}(\boldsymbol{P})$ and $\hat{a}^{\dagger}_{A\alpha}(\boldsymbol{P})$, as well as on the non-perturbed part of Hamiltonian $\bar{H}_0$. One can write the total quasiequilibrium superoperator as some non-perturbed part of $\hat{\varrho}_{\mathrm{q}}^{0}(t)$ and the part which describes interaction of quantum particles in the quasiequilibrium state. Further, we introduce the following designation:

$$\hat{\varrho}_{\mathrm{q}}(t) = \hat{\varrho}_{\mathrm{q}}^{0}(t) + \hat{\varrho}_{\mathrm{q}}'(t), \tag{3.24}$$

where

$$\hat{\varrho}_{\mathrm{q}}'(t) = -\int \mathrm{d}\boldsymbol{r}\ \beta(\boldsymbol{r};t)\int_0^1 \mathrm{d}\tau\ \hat{\varrho}_{\mathrm{q}}^{\tau}(t)\hat{H}_{\mathrm{int}}(\boldsymbol{r})(\hat{\varrho}_{\mathrm{q}}(t))^{-\tau}\hat{\varrho}_{\mathrm{q}}^{0}(t). \tag{3.25}$$

Quasiequilibrium thermo vacuum states $|\hat{\varrho}_{\mathrm{q}}(t)\rangle\rangle$ and $|\hat{\varrho}_{\mathrm{q}}^{0}(t)\rangle\rangle$ are not vacuum states for annihilation and creation superoperators $\hat{a}_{A\alpha}(\boldsymbol{P})$, $\hat{a}^{\dagger}_{A\alpha}(\boldsymbol{P})$, $\tilde{a}_{A\alpha}(\boldsymbol{P})$, $\tilde{a}^{\dagger}_{A\alpha}(\boldsymbol{P})$. But for $|\hat{\varrho}_{\mathrm{q}}^{0}(t)\rangle\rangle$ one can construct new superoperators $\hat{\gamma}_{A\alpha}(\boldsymbol{P})$, $\hat{\gamma}^{\dagger}_{A\alpha}(\boldsymbol{P})$, $\tilde{\gamma}_{A\alpha}(\boldsymbol{P})$, $\tilde{\gamma}^{\dagger}_{A\alpha}(\boldsymbol{P})$ as a linear combination of superoperators $\hat{a}_{A\alpha}(\boldsymbol{P})$, $\hat{a}^{\dagger}_{A\alpha}(\boldsymbol{P})$ and $\tilde{a}_{A\alpha}(\boldsymbol{P})$, $\tilde{a}^{\dagger}_{A\alpha}(\boldsymbol{P})$ in order to satisfy the conditions:

$$\begin{aligned}
&\hat{\gamma}_{A\alpha}(\boldsymbol{P};t)|\varrho_{\mathrm{q}}^{0}(t)\rangle\rangle = 0, \qquad && \langle\langle 1|\hat{\gamma}^{\dagger}_{A\alpha}(\boldsymbol{P};t) = 0,\\
&\tilde{\gamma}_{A\alpha}(\boldsymbol{P};t)|\varrho_{\mathrm{q}}^{0}(t)\rangle\rangle = 0, \qquad && \langle\langle 1|\tilde{\gamma}^{\dagger}_{A\alpha}(\boldsymbol{P};t) = 0.
\end{aligned} \tag{3.26}$$

To achieve this let us consider an action of annihilation superoperators $\hat{a}_{A\alpha}(\boldsymbol{P};t)$, $\tilde{a}_{A\alpha}(\boldsymbol{P};t)$ on quasiequilibrium state $|\varrho^0_{\rm q}(t_0)\rangle\rangle$:

$$\begin{aligned}\hat{a}_{A\alpha}(\boldsymbol{P};t)|\varrho^0_{\rm q}(t_0)\rangle\rangle &= \quad f_{A\alpha}(\boldsymbol{P};t-t_0)\tilde{a}^\dagger_{A\alpha}(\boldsymbol{P};t)|\varrho^0_{\rm q}(t_0)\rangle\rangle,\\ \tilde{a}_{A\alpha}(\boldsymbol{P};t)|\varrho^0_{\rm q}(t_0)\rangle\rangle &= \sigma f_{A\alpha}(\boldsymbol{P};t-t_0)\hat{a}^\dagger_{A\alpha}(\boldsymbol{P};t)|\varrho^0_{\rm q}(t_0)\rangle\rangle,\end{aligned} \tag{3.27}$$

where superoperators $\hat{a}_{A\alpha}(\boldsymbol{p};t)$, $\hat{a}^\dagger_{A\alpha}(\boldsymbol{p};t)$, $\tilde{a}_{A\alpha}(\boldsymbol{p};t)$, $\tilde{a}^\dagger_{A\alpha}(\boldsymbol{p};t)$ are in the Heisenberg representation

$$\begin{aligned}\hat{a}_{A\alpha}(\boldsymbol{P};t) &= {\rm e}^{-\frac{1}{{\rm i}\hbar}\bar{H}_0 t}\,\hat{a}_{A\alpha}(\boldsymbol{P})\,{\rm e}^{\frac{1}{{\rm i}\hbar}\bar{H}_0 t}, & \tilde{a}_{A\alpha}(\boldsymbol{P};t) &= {\rm e}^{-\frac{1}{{\rm i}\hbar}\bar{H}_0 t}\,\tilde{a}_{A\alpha}(\boldsymbol{P})\,{\rm e}^{\frac{1}{{\rm i}\hbar}\bar{H}_0 t},\\ \hat{a}^\dagger_{A\alpha}(\boldsymbol{P};t) &= {\rm e}^{-\frac{1}{{\rm i}\hbar}\bar{H}_0 t}\,\hat{a}^\dagger_{A\alpha}(\boldsymbol{P})\,{\rm e}^{\frac{1}{{\rm i}\hbar}\bar{H}_0 t}, & \tilde{a}^\dagger_{A\alpha}(\boldsymbol{P};t) &= {\rm e}^{-\frac{1}{{\rm i}\hbar}\bar{H}_0 t}\,\tilde{a}^\dagger_{A\alpha}(\boldsymbol{P})\,{\rm e}^{\frac{1}{{\rm i}\hbar}\bar{H}_0 t},\end{aligned}$$

and satisfy commutation relations:

$$\begin{aligned}\Big[\hat{a}_{A\alpha}(\boldsymbol{P};t),\hat{a}^\dagger_{B\beta}(\boldsymbol{P}';t)\Big]_\sigma &= \delta_{A,B}\delta_{\alpha,\beta}\delta(\boldsymbol{P}-\boldsymbol{P}'),\\ \Big[\tilde{a}_{A\alpha}(\boldsymbol{P};t),\tilde{a}^\dagger_{B\beta}(\boldsymbol{P}';t)\Big]_\sigma &= \delta_{A,B}\delta_{\alpha,\beta}\delta(\boldsymbol{P}-\boldsymbol{P}'),\\ \Big[\hat{a}_{A\alpha}(\boldsymbol{P};t),\tilde{a}_{B\beta}(\boldsymbol{P}';t)\Big]_\sigma &= \Big[\hat{a}^\dagger_{A\alpha}(\boldsymbol{P};t),\tilde{a}^\dagger_{B\beta}(\boldsymbol{P}';t)\Big]_\sigma = 0.\end{aligned}$$

It is necessary to note that superoperators $\hat{H}(\boldsymbol{r})$, $\hat{n}_{A\alpha}(x)$ are built on superoperators $\hat{a}_{A\alpha}(\boldsymbol{p}+\frac{\boldsymbol{q}}{2})$, $\hat{a}^\dagger_{A\alpha}(\boldsymbol{p}-\frac{\boldsymbol{q}}{2})$, $\tilde{a}_{A\alpha}(\boldsymbol{p}+\frac{\boldsymbol{q}}{2})$, $\tilde{a}^\dagger_{A\alpha}(\boldsymbol{p}-\frac{\boldsymbol{q}}{2})$. Therefore, for convenience here a unit denotion was introduced for arguments like $\boldsymbol{P}=\boldsymbol{p}\pm\frac{\boldsymbol{q}}{2}$. This should be taken into account in further calculations where obvious expressions are needed.

We can introduce new operators $\hat{\gamma}_{A\alpha}(\boldsymbol{P};t)$, $\hat{\gamma}^\dagger_{A\alpha}(\boldsymbol{P};t)$, $\tilde{\gamma}_{A\alpha}(\boldsymbol{P};t)$, $\tilde{\gamma}^\dagger_{A\alpha}(\boldsymbol{P};t)$ via superoperators $\hat{a}_{A\alpha}(\boldsymbol{P};t)$, $\hat{a}^\dagger_{A\alpha}(\boldsymbol{P};t)$, $\tilde{a}_{A\alpha}(\boldsymbol{P};t)$, $\tilde{a}^\dagger_{A\alpha}(\boldsymbol{P};t)$ in the following way:

$$\begin{aligned}\hat{\gamma}_{A\alpha}(\boldsymbol{P};t) &= \sqrt{1+\sigma n_{A\alpha}(\boldsymbol{P};t,t_0)}\left[\hat{a}_{A\alpha}(\boldsymbol{P};t)-\frac{n_{A\alpha}(\boldsymbol{P};t,t_0)}{1+\sigma n_{A\alpha}(\boldsymbol{P};t,t_0)}\tilde{a}^\dagger_{A\alpha}(\boldsymbol{P};t)\right],\\ \tilde{\gamma}^\dagger_{A\alpha}(\boldsymbol{P};t) &= \sqrt{1+\sigma n_{A\alpha}(\boldsymbol{P};t,t_0)}\left[\tilde{a}^\dagger_{A\alpha}(\boldsymbol{P};t)-\sigma\hat{a}_{A\alpha}(\boldsymbol{P};t)\right].\end{aligned} \tag{3.28}$$

Relations (3.28) satisfy conditions (8.26). Here

$$\begin{aligned}&n_{A\alpha}(\boldsymbol{p},\boldsymbol{q};t,t_0) = n_{A\alpha}(\boldsymbol{P};t,t_0), = \langle\langle 1|\tilde{a}^\dagger_{A\alpha}(\boldsymbol{P};t)\tilde{a}_{A\alpha}(\boldsymbol{P};t)|\varrho^0_{\rm q}(t_0)\rangle\rangle =\\ &\langle\langle 1|\tilde{a}^\dagger_{A\alpha}(\boldsymbol{p}-\frac{\boldsymbol{q}}{2};t)\tilde{a}_{A\alpha}(\boldsymbol{p}+\frac{\boldsymbol{q}}{2};t)|\varrho^0_{\rm q}(t_0)\rangle\rangle,\end{aligned}$$

is a quasiequilibrium distribution function of A-particle coupled states in momentum space $\boldsymbol{p}$, $\boldsymbol{q}$, which is calculated with the help of quasiequilibrium thermo vacuum state vector $\varrho^0_{\rm q}(t_0)\rangle\rangle$ (3.22). Function $f_{A\alpha}(\boldsymbol{P};t-t_0)$ in formulae (3.27) is connected with $n_{A\alpha}(\boldsymbol{P};t,t_0)$ by the relation

$$f_{A\alpha}(\boldsymbol{P};t-t_0) = \frac{n_{A\alpha}(\boldsymbol{P};t,t_0)}{1+\sigma n_{A\alpha}(\boldsymbol{P};t,t_0)}.$$

Superoperators $\hat{\gamma}_{A\alpha}(\boldsymbol{P};t)$ and $\tilde{\gamma}_{A\alpha}(\boldsymbol{P};t)$, $\hat{\gamma}^\dagger_{A\alpha}(\boldsymbol{P};t)$ and $\tilde{\gamma}^\dagger_{A\alpha}(\boldsymbol{P};t)$ satisfy the “canonical” commutation relations:

$$\begin{aligned}\Big[\hat{\gamma}_{A\alpha}(\boldsymbol{P};t),\hat{\gamma}^\dagger_{B\beta}(\boldsymbol{P}';t)\Big]_\sigma &= \delta_{A,B}\delta_{\alpha,\beta}\delta(\boldsymbol{P}-\boldsymbol{P}'),\\ \Big[\tilde{\gamma}_{A\alpha}(\boldsymbol{P};t),\tilde{\gamma}^\dagger_{B\beta}(\boldsymbol{P}';t)\Big]_\sigma &= \delta_{A,B}\delta_{\alpha,\beta}\delta(\boldsymbol{P}-\boldsymbol{P}'),\\ \Big[\hat{\gamma}_{A\alpha}(\boldsymbol{P};t),\tilde{\gamma}_{B\beta}(\boldsymbol{P}';t)\Big]_\sigma &= \Big[\hat{\gamma}^\dagger_{A\alpha}(\boldsymbol{P};t),\tilde{\gamma}^\dagger_{B\beta}(\boldsymbol{P}';t)\Big]_\sigma = 0.\end{aligned} \tag{3.29}$$

Inversed transformations to superoperators $\hat{a}_{A\alpha}(\boldsymbol{P};t)$, $\tilde{a}^{\dagger}_{A\alpha}(\boldsymbol{P};t)$ are easily obtained from (3.28):

$$\hat{a}_{A\alpha}(\boldsymbol{P};t) = \sqrt{1+\sigma n_{A\alpha}(\boldsymbol{P};t,t_0)}\left[\hat{\gamma}_{A\alpha}(\boldsymbol{P};t) + \frac{n_{A\alpha}(\boldsymbol{P};t,t_0)}{1+\sigma n_{A\alpha}(\boldsymbol{P};t,t_0)}\tilde{\gamma}^{\dagger}_{A\alpha}(\boldsymbol{P};t)\right],$$
$$\tilde{a}^{\dagger}_{A\alpha}(\boldsymbol{P};t) = \sqrt{1+\sigma n_{A\alpha}(\boldsymbol{P};t,t_0)}\left[\tilde{\gamma}^{\dagger}_{A\alpha}(\boldsymbol{P};t) + \sigma\hat{\gamma}_{A\alpha}(\boldsymbol{P};t)\right]. \qquad (3.30)$$

$\hat{\gamma}_{A\alpha}(\boldsymbol{P};t)$, $\hat{\gamma}^{\dagger}_{A\alpha}(\boldsymbol{P};t)$, $\tilde{\gamma}_{A\alpha}(\boldsymbol{P};t)$, $\tilde{\gamma}^{\dagger}_{A\alpha}(\boldsymbol{P};t)$ could be defined as some operators of annihilation and creation of A-quasiparticle coupled states, for which quasiequilibrium thermo vacuum state $|\varrho^0_{\rm q}(t_0)\rangle\rangle$ (3.22) is a vacuum state. In such a way, we obtained relations of dynamical reflection of superoperators $\hat{a}_{A\alpha}(\boldsymbol{P};t)$, $\hat{a}^{\dagger}_{A\alpha}(\boldsymbol{P};t)$, $\tilde{a}_{A\alpha}(\boldsymbol{P};t)$, $\tilde{a}^{\dagger}_{A\alpha}(\boldsymbol{P};t)$ to new superoperators of "quasiparticles" $\hat{\gamma}_{A\alpha}(\boldsymbol{P};t)$, $\hat{\gamma}^{\dagger}_{A\alpha}(\boldsymbol{P};t)$, $\tilde{\gamma}_{A\alpha}(\boldsymbol{P};t)$, $\tilde{\gamma}^{\dagger}_{A\alpha}(\boldsymbol{P};t)$.

4. Conclusions

A set of transport equations (3.12), (3.13) together with dynamical reflections (3.28), (3.30) of superoperators in the thermo field space constitute the basis for a consistent description of the kinetics and hydrodynamics of a dense quantum system with strongly coupled states. Both strongly and weakly nonequilibrium processes of a nuclear matter can be investigated using this approach, in which the particle interaction is characterized by strongly coupled states, taking into account theirs nuclear nature [1, 2, 8, 9].

Another problem in the description of quantum kinetic processes of nuclear collisions should be noted. It is connected with the construction of quantum kinetic equations for small times with taking account initial states and non-Markovian memory effects. One approach to obtain such kinetic equations is developed on the basis of mixed Green functions in recent papers by Morozov, Röpke and others [39, 40]. In our approach the problem of initial states is connected with the quasiequilibrium thermo vacuum state $|\varrho_{\rm q}(t_0)\rangle\rangle$ at the initial time t_0. Non-Markovian memory effects are described here by the generalised memory functions $\varphi^{A\alpha B\beta}_{nn}$, $\varphi^{A\alpha}_{nH}$, $\varphi^{B\beta}_{Hn}$, φ_{HH} (3.14)–(3.17), correspondingly. These functions are calculated with the help of $|\varrho(t)\rangle\rangle_{t=t_0}$ or $|\varrho_{\rm q}(t_0)\rangle\rangle$ (see [41]) and take into account both oneparticle and manyparticle processes of energy transfer in a system.

In the next paper we will consider in detail weakly nonequilibrium case and obtain generalised closed transfer equations for Wigner function $f_{A\alpha}(x;t)$ and mean energy density $\langle\hat{H}(\boldsymbol{r})\rangle^t$ for quantum system with coupled states. We will suggest also one way of calculation of generalised memory functions in thermo field representation. It allows to analyse spectra of time correlation functions like "density-density", "current-current" and "energy-energy" as well as generalised transport coefficients for quantum systems.

It is much sequential to describe investigations of kinetic and hydrodynamic processes of a nuclear matter on the basis of quark-gluon interaction. The quantum relativistic theory of kinetic and hydrodynamic processes has its own problems and experiences its impetuous formation [1, 2, 3, 4, 5, 6]. In the next papers we will apply our approach to describe kinetic and hydrodynamic processes of a quark-gluon plasma.

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